Some aspects of the asymptotic behavior of solutions to the Navier-Stokes equations

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Abstract

We study some aspects of asymptotic behavior of strong global solutions of the homogeneous Navier-Stokes equations (see also [1]). Ω is a domain in $\mathbf{R}^3$ with uniform C^3 boundary, A is the Stokes operator and $\{E_\lambda; \lambda \geq 0\}$ is the resolution of identity of A. $|||\cdot|||_\beta = ||\cdot|| + ||A^\beta \cdot||$ denotes the graph norm and w is a strong global solution, $w \neq 0$.

Theorem 2 *Suppose that there exist $\kappa < -1$ and $C > 0$ such that $||w(t)|| \leq Ct^\kappa$ for every $t \geq 1$. Let $C(\beta) = \limsup_{t\to\infty} ||A^\beta w(t)||/||w(t)||$. Then $0 \leq C(\beta) < \infty$ and $C(\beta)^{1/\beta}$ is independent of $\beta \in (0,1)$. Let $\lambda = C(\beta)^{1/\beta}$. If $0 \leq \lambda_1 < C(\beta)^{1/\beta} < \lambda_2 < \infty$ then*

$$\lim_{t\to\infty} \frac{|||E_{\lambda_2} w(t) - E_{\lambda_1} w(t)|||_\beta}{|||w(t)|||_\beta} = 1 \text{ and } \lim_{t\to\infty} \frac{||E_{\lambda_2} w(t) - E_{\lambda_1} w(t)||}{||w(t)||} = 1. \quad \blacksquare$$

Let Ω be a smooth bounded domain, $\{\lambda_j\}_{j=1}^\infty$ be the non-decreasing sequence of eigenvalues of A and let w_j be the eigenfunction of A associated with λ_j. If $w = \sum_j \alpha_j w_j$, we put $P_n w = \sum_j^n \alpha_j w_j$ for every $n \in N$.

Theorem 3 *There exists a unique $n = n(w) \in N$ such that $\lambda_n < \lambda_{n+1}$ and*

$$\lim_{t\to\infty} \frac{||A^\beta (I - P_n) w(t)||}{||A^\beta P_n w(t)||} = 0, \ \forall \beta \in [0, 5/4).$$

If $\lambda_n > \lambda_1$ and $k = k(w)$ denotes the largest natural number such that $\lambda_k < \lambda_n$ then

$$\lim_{t\to\infty} \frac{||A^\beta (P_n - P_k) w(t)||}{||A^\beta P_k w(t)||} = \infty, \ \forall \beta \in [0, 5/4). \quad \blacksquare$$

If $n \in N$ and $\lambda_n < \lambda_{n+1}$ then there exists w such that $n = n(w)$. The set of initial conditions of such solutions contains a smooth manifold in the space $D(A^\gamma)$, $\gamma \in (3/4, 1)$.

Keywords: Navier-Stokes equations, asymptotic behavior, strong global solution.

References

[1] Z.Skalák, On asymptotic dynamics of solutions of the homogeneous Navier-Stokes equations, *Nonlinear Analysis, Theory, Methods and Applications*, paper in press.