Local space-time regularity criteria for weak solutions of the Navier-Stokes equations beyond Serrin's condition

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Abstract

Consider a weak solution $u = u(t, x)$ of the Navier-Stokes equations satisfying the localized energy inequality for a general domain $\Omega \subset I\!R^3$ on the time interval $[0, \infty)$. Then the local condition of Caffarelli, Kohn and Nirenberg (1982) requiring smallness of the $L^3(L^3)$-norm of u *and* of a suitable norm of the pressure on a space-time cylinder $Q_r(t, x) = (t - r^2, t) \times B_r(x)$ guarantees local regularity. A related condition on ∇u (or on $\operatorname{curl} u$ only, see Wolf (2006)) needs the smallness of the *limes superior* of weighted $L^2(L^2)$-norms on $Q_r(t, x)$ as $r \to 0$ to imply the same result.

In this talk we present a new local regularity criterion beyond Serrin's barrier involving a smallness condition on u only. To be more precise, if for some $r > 0$

$$\Big(\int_{t-r^2}^{t} \|u\|^s_{L^q(B_r(x))} d\tau \Big)^{1/s} \le \varepsilon_* r^{\frac{2}{s}+\frac{3}{q}-1},$$

where $1 \le \frac{2}{s} + \frac{3}{q} \le 1 + \frac{1}{q}$, $2 < s \le q < \infty$, and $\varepsilon_* > 0$ is an absolute constant, then u is regular on $Q_{r/2}(t, x)$. In the limit case $s = q = 4$ the same result holds for an arbitrary weak solution. The main idea is the *theory of very weak solutions* of the Navier-Stokes equations introduced by Amann and recently extended by Farwig, Kozono, Schumacher, Simader, Sohr and Galdi.

Keywords: weak solutions, local regularity, Serrin's condition, very weak solutions