Compressible Navier-Stokes equations with non-constant viscosities

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Abstract

The aim of this talk is to present mathematical results on global existence of solutions (weak and strong) to the time dependent compressible Navier-Stokes equations with non-constant viscosities without symmetry hypothesis on the domain, without hypothesis on the size of the initial data and without hypothesis such as initial data close to equilibrium. The Navier-Stokes equations for viscous compressible and heat conducting fluids reads

$$\partial_t\rho + \mathrm{div}(\rho u) = 0, \tag{1}$$

$$\partial_t(\rho u) + \mathrm{div}(\rho u \otimes u) = \mathrm{div}\sigma + \rho f, \tag{2}$$

$$\partial_t(\rho E) + \mathrm{div}(\rho u H) = \mathrm{div}((\sigma + pI)\cdot u) + \mathrm{div}(\kappa\nabla\theta) + \rho f\cdot u, \tag{3}$$

$$E = e + \frac{|u|^2}{2}, \quad H = h + \frac{|u|^2}{2}, \quad h = e + \frac{p}{\rho},$$

where $u \in R^d$ denotes the velocity field, ρ the density, κ the thermic conductivity, σ the stress tensor, $p$the pressure, e the specific internal energy and h the specific enthalpy. The specific total energy is denoted E and the specific total enthalpy H. Finally the external forces are given by f. The flow is assumed to be newtonian that means there exists two viscosity coefficients μ and λ such that

$$\sigma = 2\mu D(u) + (\lambda\mathrm{div}u - p)\,\mathrm{Id}, \tag{4}$$

where $D(u)$ denotes the strain tensor defined as the symmetric part of the velocity gradient ∇u, namely $D(u) = (\nabla u + {}^t\nabla u)/2$. The pressure p and the internal energy e are given functions of the density ρ and temperature θ with a thermodynamical compatibility condition. At first, we will present results on the isentropic Navier-Stokes equations namely without equation (3) and with a pressure law p and viscosities λ, μ depending on the density ρ. In a second

part, we will discuss on viscous compressible and heat conducting fluids namely the complete system written before. We will present results with density or temperature dependent viscosities. We hope by this talk to present an overview of the subject with interesting open problems and possible extensions to other systems: see for instance MHD, Born-Infeld models by R. Sart.

Keywords: Compressible Navier-Stokes equations, global existence, mathematical entropy, isentropic flow, heat conducting flow.

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