On the motion of floating bodies

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Abstract

In two papers from 1949 and 1950 Fritz John [1] investigated the motion of a mechanical system consisting of a heavy liquid and a partly immersed body. He assumed the liquid to be incompressible and to have an irrotational motion; the free surface of the liquid extends to infinity in all directions, and it is determined as a surface of constant pressure. Here we shall investigate a similar problem; the fluid is viscous and incompressible and the free surface is governed by surface tension; moreover, we assume that it meets the body at a right angle. We show existence of a classical solution for various configurations. In the first case we consider a circular cylinder, partly filled with liquid, and a ball B (some other body with axial symmetry) that has smaller density than the fluid. If B is positioned on the axis of the cylinder and rotates with small angular velocity there exists an equilibrium configuration that is axially symmetric. The weight of the body and the force exerted on it by the motion of the fluid determine its position. The existence proof is based on a method of D. Sattinger [2] who investigated the flow in a cylinder when a rod is inserted in the center. The velocity is of class $C^{2,\alpha}$ up to the boundary. The regularity in the ridge where the capillary surface meets the body is due to the fact that Dirichlet conditions are prescribed on the fixed boundary and a Neumann-type condition on the free surface; necessary and sufficient conditions for regularity in such cases are due to E.A. Volkov [3].

The case of an unbounded fluid can be treated similary. However, if a body is driven by a flow the torque exerted on it no longer vanishes. This causes a rotation of the body such that the contact line is moving. In order to prevent this we assume that the body is hollow inside and that we can move masses about such that we establish a center of mass that excludes this rotation. In a similar problem this device was introduced by H.F. Weinberger [4].

Keywords: Navier-Stokes equations, free-boundary problem.

References

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