Isotropic and anisotropic weighted Sobolev spaces for the Oseen equations

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Abstract

We are interested here by the study of the two or three-dimensional Oseen problem (see Babenko, Farwig, Finn, Galdi, Kracmar, Novotny, Pokorny, Simader, Sohr, ...). Thanks to Marcinkiewicz interpolation's theorem and to Sobolev embeddings, we give in a first part a new proof of Lizorkin theorem concerning the L^p continuity's properties of the Oseen potential. In a second part, we prove existence and uniqueness results in weighted Sobolev spaces for the Oseen problem in $\mathbb{R}^d$ or in exterior domains of $\mathbb{R}^d$, with $d = 2$ or 3:

$$-\nu\Delta \boldsymbol{u} + \frac{\partial \boldsymbol{u}}{\partial x_1} + \nabla\pi = \boldsymbol{f}, \quad \text{div } \boldsymbol{u} = g \quad \text{in} \quad \Omega, \quad \boldsymbol{u} = \boldsymbol{u}_* \quad \text{on} \quad \Gamma. \tag{1}$$

Our study is based on a isotropic or anisotropic weighted L^p theory for any real p such that $1 < p < \infty$. In particular, we show that, under optimal assumptions on the data $\boldsymbol{f}$, g and $\boldsymbol{u}_*$, the problem (0.1) has a solution $(\boldsymbol{u}, \pi)$ satisfying $\nabla \boldsymbol{u} \in L^p(\Omega)$ and $\pi \in L^p(\Omega)$, for any $1 < p < \infty$. To our knowledge, the case $1 < p \leq 3/2$ was an *open question.* Our analysis is mainly based on the principle that the linear exterior problems can be solved by combining their properties in the whole space $\mathbb{R}^d$ and their properties in bounded domains.

Keywords:

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