Some extensions of Serrin's regularity condition for the Navier-Stokes equations

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Abstract

We consider several possibilities to extend Serrin's regularity condition

$$\frac{2}{s} + \frac{3}{q} = 1$$

for weak solutions $\mathbf{u}$ of the Navier-Stokes equations:

i) The Lebesgue spaces withs exponents s, q in Serrin's criterion can be replaced by larger spaces like Lorentz spaces or Sobolev spaces of negative order.

ii) Under some smallness conditions on $\mathbf{u}$ we are able to extend Serrin's criterion to smaller exponents s, q such that $\frac{2}{s} + \frac{3}{q} > 1$.

iii) Serrin's (global) condition can be replaced by leftward local conditions at each time.

iv) The Lebesgue space in time direction can be replaced by some Hölder space which also extends Serrin's condition in a certain sense.

v) There are several further regularity conditions which are based on energy quantities.