Further applications of the Cosserat spectrum

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Abstract

Let $G \subset \mathbb{R}^n$ $(n \geq 2)$ be a bounded domain. A number $\lambda \in \mathbb{R}$ is called a Cosserat-eigenvalue if there exists $0 \neq \underline{u} \in \left[C^2(G) \cap C^0(\bar{G})\right]^n$ such that

$$\Delta \underline{u} = \lambda \nabla \operatorname{div} \underline{u} \text{ in } G, \quad \underline{u}\,|_{\partial G} = 0. \tag{1}$$

We consider weak L^q-solutions $(1 < q < \infty)$ of (1). For this purpose let

$$L_0^q(G) := \left\{ p \in L^q(G) : \int_G p dy = 0 \right\}$$

If $\partial G \in C^1$ for $p \in L_0^q(G)$ there exists a unique $\underline{v} \in \underline{H}_0^{1,q}(G) := H_0^{1,q}(G)^n$ such that

$$\langle \nabla \underline{u}, \nabla \underline{\phi} \rangle = \langle p, \operatorname{div} \underline{\phi} \rangle \qquad \forall \underline{\phi} \in \underline{H}_0^{1,q'}(G) \tag{2}$$

where $q' = \frac{q}{q-2}$. Then $0 \neq \underline{u} \in \underline{H}_0^{1,q}(G)$ is a weak solution of (1) if (2) holds with $p = \lambda \cdot \operatorname{div} \underline{u}$. Let $Z_q : L_0^q(G) \to L_0^q(G)$ be defined by $Z_q(p) := \operatorname{div} \underline{u}$, where $\underline{u}$ is the solution of (2). Let $B_0^q(G) := \{p \in L_0^q(G) : \Delta p = 0\}$. The decisive fact is that the operator

$$Z_q - \frac{1}{2} I : B_0^q(G) \to B_0^q(G)$$

is compact (St. Weyers, [2]). This fact has a lot of applications, e.g. very simple proofs of existence for Stokes' and Lamé's equations [1]. Analogous results can be proved for exterior domains if the "classical" Sobolev spaces $H_0^{1,q}(G)$ are replaced by the "bigger" spaces $\hat{H}_\bullet^{1,q}(G)$.

Keywords: Cosserat spectrum, divergence equation, Stokes equation, Lamé's equation.

References

[1] C. G. Simader, An operator related to the Cosserat spectrum. Applications, *Analysis (Munich)* 26, 169-198, 2006.

[2] St. Weyers, L^q-solutions to the Cosserat spectrum in bounded and exterior domains, *Analysis (Munich)* 26, 85-167, 2006