Stability results related to MHD models for compressible fluids

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Abstract

The aim of this talk is to present some stability results for models describing the motion of a compressible fluid under magnetic influence. On the one hand, we follow the works of P.-L. Lions and E. Feireisl on Navier-Stokes equations for compressible fluids to get the existence of global weak solutions of a classical MHD model with constant viscosity in the adiabatic case $P(\varrho) = \varrho^\gamma$, $\gamma > 3/2$. On the other hand, for models with density-dependent viscosities, we take advantage of the BD entropy recently introduced by D. Bresch and B. Desjardins. We first talk about the nonlinear form of electromagnetism given by the 3D Born-Infeld system. We get the stability of weak solutions for the complete Augmented Born-Infeld model with temperature equation and the perfect gas law $P(\varrho, \theta) = r\varrho\theta$. The main tool of the strategy given by D. Bresch and B. Desjardins consist in obtaining a better control of the density through the estimate

$$\int_0^T \int_\Omega \frac{|\nabla\mu(\varrho)|^2}{\varrho} \leq C,$$

for a viscosity μ separetly defined for small and large densities. This way can also be followed to study a 2D MHD model of a fluid-fluid diffuse interface for quite different conditions especially about the viscosity and resistivity profiles.

Keywords: Compressible fluids, magneto-hydro-dynamics, born-infeld.

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