Streaming flow effects in the nearly inviscid faraday instability

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Abstract

We study the weakly nonlinear evolution of Faraday waves in a two dimensional version of a vertically vibrating annular container. These waves, named after Faraday [1], have attracted a great deal of attention, especially because of the rich variety of non-linear pattern forming phenomena promoted by the Faraday instability ([2, 3, 4]). Unfortunately, current theoretical approaches fail to appropriately explain essential issues associated with the behavior beyond threshold, particularly in the singular limit of small viscosity. The object of the present work is precisely to analyze the small viscosity limit, where the evolution of the surface waves is coupled to a non-oscillatory streaming flow that develops in the bulk of the container and is driven by the boundary layers at the container walls and free surface. A system of equations is derived for the coupled slow evolution of the spatial phase of the surface wave and the streaming flow. These equations are numerically integrated to show that the simplest reflection symmetric steady state (the usual array of counter-rotating eddies below the surface wave) becomes unstable for realistic values of the parameters. The new states include limit cycles, steadily travelling waves (which are standing in a moving reference frame), and some more complex attractors [5]. We also consider the effect of surface contamination (that is likely to be present in water, as in [6], unless care is taken in the experimental set-up), modelled by Marangoni elasticity with insoluble surfactant, in promoting drift instabilities in spatially uniform standing Faraday waves. It is seen that contamination enhances drift instabilities that lead to various steadily propagating travelling waves and (both standing and propagating) oscillatory patterns. In particular, steadily propagating waves appear to be quite robust, as in the experiment by Douady *et al.*[6]

Keywords: Faraday waves, streaming flow, weakly nonlinear analysis, surface contamination, marangoni elasticity.

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