Onsager's conjecture and a model for turbulence [1]

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Abstract

In 1949 Onsager conjectured that in 3 dimensional turbulent flows energy dissipation might exist even in the limit of vanishing viscosity. He suggested that an appropriate mathematical description of turbulent flows (in the inviscid limit) might be given by weak solutions of the Euler equation that are not regular enough to conserve energy. According to this view, non-conservation of energy in a turbulent flow might occur not only from viscous dissipation, but also from lack of smoothness of the velocity. Specifically, Onsager conjectured that weak solutions of the Euler equation with Holder continuity exponent $h > 1/3$ conserve energy and that turbulent or anomolous dissipation occurs when h is less or equal to 1/3. In the past decade a number of results have been obtained giving function spaces in which the energy of (putative) weak solutions to the Euler equation is conserved.

We discuss results for a shell type dyadic model for the forced Euler equations. By construction, all the interactions in the model system are local and energy cascades strictly to higher wave numbers. There is a unique fixed point for the system which is a global attractor. Onsager's conjecture is confirmed for the model in both directions, i.e. solutions with bounded $H^{5/6}$ norm satisfy the energy balance condition and turbulent dissipation occurs for all solutions when the $H^{5/6}$ norm becomes unbounded, which happens in finite time.

We conclude with some recent results for the 3D Euler equation itself: we obtain the sharp function space for conservation of energy and we show that the energy flux is controlled by local interactions, thus confirming the physical intuition that underlies much of turbulence theory.

Keywords: Euler equations, anomalous dissipation, Onsager's conjecture, dyadic model.

[1] This is joint work with Peter Constantin, Alexey Cheskidov, Natasa Pavlovic and Roman Shvydkoy.